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On the transversal vibrations of a conveyor belt: Applicability of simplified models

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Abstract

In this paper, some aspects in the analysis of transversal oscillations of conveyor belts will be discussed. In particular the use of finite or infinite mode-representations to describe the oscillations will be discussed, and the applicability of the underlying partial differential equations will be taken into account.

1. Introduction

For a long time all kinds of aspects of conveyor-belt oscillations have been studied. Recently, it has been pointed out in Refs. [1–3] that it is not always mathematically correct to truncate the infinite mode-representation for the oscillations to a single or a few modes of oscillations. Usually partial differential equations are used to describe these oscillations. For non-resonant problems the truncation method can usually be applied successfully. For a resonant problem, however, it has been explicitly shown in Refs. [1–3] that the truncation method (and so, a finite mode representation for the solution of the problem) can lead in certain cases to results which describe wrong internal mode-interactions, and which give rise to wrong resonance frequencies. Since the (in)stability of a system is usually determined by the internal mode-interactions and the resonance frequencies it follows for instance from Refs. [1–3] that one has to be careful in applying the truncation method.

On the other hand, there exist a lot of engineering approaches (see the list of references in Ref. [2,3]) that only use a single or a few modes to describe these oscillations. The aim of this paper is to contribute to filling the gap between the above mentioned approaches, that is, the gap between an infinite mode-representation or a finite one. Asymptotic techniques (as for instance described in Ref. [4]) will be used to illustrate how this gap most likely can be filled, and a new approach will be proposed.

In the new approach the applicability of the string model, the stretched beam model, and the beam model will be taken into account. That is, for the lower frequencies (and oscillation mode numbers) a perturbed

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string model is appropriate, for the higher frequencies a perturbed beam model is applicable, and for the intermediate frequencies a perturbed stretched beam model is the most appropriate model to describe the dynamics of the conveyor belt correctly. Each submodel has its own physical and mathematical properties. One of the mathematical properties is related to the applicability of the truncation method. So, instead of using only one model (string, or beam, or beam–string) a combination model is proposed, where the model equations depend on the frequencies and on the vibration mode numbers.

The paper is organized as follows. In Section 2 of this paper a problem for a conveyor belt with an internal resonance will be formulated, and in Section 3 the interplay between different small parameters and the interplay between different modes will be discussed. For the lower frequencies and for the higher frequencies it will turn out that a string-like model and a beam-like model, respectively, are applicable. These models will be discussed briefly in Section 4 and in Section 5 of this paper. The stretched beam model is already discussed in Ref. [2]. Finally, in Section 6 of this paper some conclusions will be drawn.

2. The equation of motion

The simplest mechanical model for a conveyor belt is a 1D-model in the spatial variable x (see Figs. 1 and 2). In this framework the equation of motion may be written as follows:

$$u_{tt} + 2Vu_{xt} + V_t u_x + (V^2 - c^2)u_{xx} + \frac{EI}{\rho A} u_{xxxx} = 0, \tag{1}$$

where $u(x,t)$ is the transversal displacement; V is the time-varying belt speed; c is the wave speed due to a pretension in the belt; E is Young's modulus; I is the moment of inertia with respect to the belt middle plane, $I = L_1 h^3/12$ (see Fig. 2); ρ is the mass density of the belt; A is the area of the belt cross-section, $A = L_1 h$ (see Fig. 2); x is the space coordinate, and t is time.

The following simply supported boundary conditions are assumed:

$$u = u_{xx} = 0 \quad \text{for } x = 0,\ \pi L, \tag{2}$$

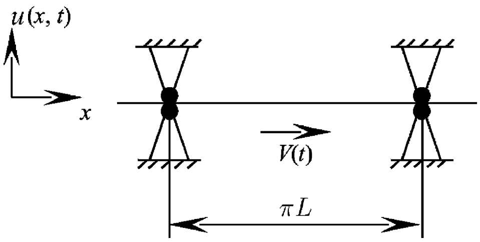

Fig. 1. A 1D model for conveyor belt.

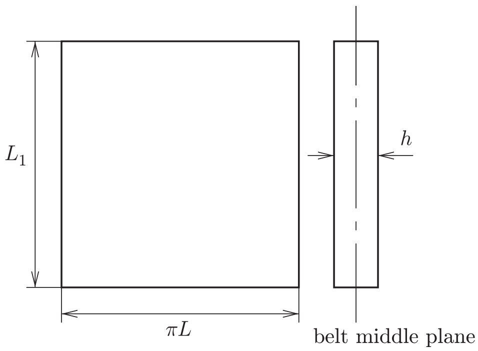

Fig. 2. The conveyor belt configuration.

where πL is the distance between the pulleys, and the initial conditions take the following form:

$$u = \varphi(x), \quad u_t = \psi_1(x) \quad \text{for } t = 0. \tag{3}$$

It is supposed that the belt velocity $V(t)$ is given by

$$V(t) = \varepsilon_1(V_0 + \alpha \sin(\Omega t)), \tag{4}$$

where $0 \leqslant \varepsilon_1 \ll 1$; V_0, α and Ω are constants with $|V_0| \gg |\alpha|$.

Eq. (1) in non-dimensional form becomes:

$$u_{\tau\tau} - c_1^2 u_{\xi\xi} + \varepsilon_2^2 u_{\xi\xi\xi\xi} = -\varepsilon u_\xi \cos\tau - 2\varepsilon(V_1 + \sin\tau)u_{\xi\tau} - \varepsilon^2(V_1 + \sin\tau)^2 u_{\xi\xi}, \tag{5}$$

where $\tau = \Omega t$; $\xi = x/L$; $c_1^2 = c^2/(L^2\Omega^2)$; $V_1 = V_0/\alpha$; $\varepsilon = \varepsilon_1\alpha/(L\Omega)$; $\varepsilon_2^2 = EI/(\rho A L^4 \Omega^2)$. For real problems $|\varepsilon| \ll 1$; $\varepsilon_2^2 \ll 1$.

In the new variable the boundary and the initial conditions take the following form:

$$u = 0 \quad \text{for } \xi = 0,\ \pi, \tag{6}$$

$$u_{\xi\xi} = 0 \quad \text{for } \xi = 0,\ \pi, \tag{7}$$

$$u = \varphi(\xi), \quad u_\tau = \psi(\xi) \quad \text{for } \tau = 0, \tag{8}$$

where $\psi(\xi) = \psi_1/\Omega$. To make our ideas more clear it will be assumed for simplicity that $c_1^2 \sim 1$, $V_1 = 2$.

By taking $c_1^2 = 1$ and $\varepsilon_2^2 \ll 1$ it is additionally assumed that an internal resonance occurs at the lowest eigenfrequency of the system (see also Ref. [1]).

3. The interplay of small parameters

The main feature in Eq. (5) is the presence of two small parameters, ε and ε_2^2. The analysis strongly depends upon their relations. Generally speaking, one has to consider (at least) three different cases:

$$(a)\ \varepsilon_2^2 \ll \varepsilon; \quad (b)\ \varepsilon_2^2 \sim \varepsilon; \quad (c)\ \varepsilon_2^2 \gg \varepsilon.$$

The analysis will be restricted to case (a), because this case is closely related to relevant problems for the conveyor belt. Further it will be assumed that $\varepsilon_2 \sim \varepsilon$.

During the analysis one also has to take into account the so-called index of the variation of the function u. The index of variation of a function $u(\xi, \varepsilon)$ as $\varepsilon \to 0$ is the number γ such that [4,5]:

$$u_\xi \sim \varepsilon^{-\gamma} u.$$

The essence of the index of the variation of the function one can easily see from the following simple example. When

$$u = A\sin(k\xi), \quad \text{then } u_\xi = Ak\cos(k\xi).$$

For $k \sim \varepsilon^{-\gamma}$ it follows that

$$u \sim A \quad \text{and} \quad u_\xi \sim A\varepsilon^{-\gamma}.$$

We also introduce the index of variation δ:

$$u_\tau = \varepsilon^{-\delta} u.$$

Comparing the second term and the third term in the left-hand side of Eq. (5) one obtains:

(a) $0 \leqslant \gamma \ll 1$, $\delta = \gamma$, then the third term in Eq. (5) can be neglected with respect to the second term. Then one obtains in the first approximation the string model

$$u_{\tau\tau} - c_1^2 u_{\xi\xi} = -\varepsilon f_1(\xi, \tau, u),$$

where

$$f_1(\xi, \tau, u) = -u_\xi \cos\tau - 2(2 + \sin\tau)u_{\xi\tau}.$$

(b) $\gamma = \delta = 1/2$, then one obtains in the first approximation the string model with another right-hand part

$$u_{\tau\tau} - c_1^2 u_{\xi\xi} = \varepsilon f_2(\xi, \tau, u),$$

where

$$f_2(\xi, \tau, u) = -2(2 + \sin\tau)u_{\xi\tau} - \varepsilon_2^2 \varepsilon^{-1} u_{\xi\xi\xi\xi}.$$

(c) $\gamma = \delta = 1$, then all terms in the left-hand side of Eq. (5) have the same order (the stretched beam model)

$$u_{\tau\tau} - c_1^2 u_{\xi\xi} + \varepsilon_1^2 u_{\xi\xi\xi\xi} = -\varepsilon f_3(\xi, \tau, u),$$

where

$$f_3(\xi, \tau, u) = -2(2 + \sin\tau)u_{\xi\tau}.$$

(d) $\gamma = \delta > 1$, then the second term in Eq. (5) can be neglected with respect to the third (the beam model)

$$u_{\tau\tau} + \varepsilon_2^2 u_{\xi\xi\xi\xi} = f_4(\xi, \tau, u),$$

where

$$f_4(\xi, \tau, u) = c_1^2 u_{\xi\xi}.$$

For cases (c) and (d) the boundary conditions have the form (6) and (7). For the string model one must use only condition (6).

Now the applicability of the above mentioned simplified models can be determined by considering

$$u_{\tau\tau} - c_1^2 u_{\xi\xi} + \varepsilon_2^2 u_{\xi\xi\xi\xi} = 0 \tag{9}$$

subject to the boundary conditions (6) and (7). To determine the eigenfrequencies of this problem the following ansatz will be used

$$u = e^{i\omega t} \sin(k\xi), \quad i = \sqrt{-1}.$$

It then follows from Eqs. (9), (6) and (7) that

$$\omega = k\sqrt{c_1^2 + \varepsilon_2^2 k^2}. \tag{10}$$

For the string model one will find that

$$\omega = kc_1 \tag{11}$$

and for the beam model

$$\omega = \varepsilon_2 k^2. \tag{12}$$

Numerical results for $c_1^2 = 0.99$, $\varepsilon_2^2 = 0.01$ show that with a 5% error in the frequency ω (which is typical for engineering problems) the string model is approximately valid for $0 \leqslant k \leqslant 7$, the beam model for $k \geqslant 22$, and the stretched beam model for $8 \leqslant k \leqslant 21$. Application of the beam model is of course also restricted by the 3D theory of elasticity, that is, additionally it should be assumed that $\pi L/k \gg h$ and $L_1 \gg h$. From now on it will be assumed that for $1 \leqslant k \leqslant k_1$ the string model can be used, that for $k_1 + 1 \leqslant k \leqslant k_2 - 1$ the stretched beam model can be applied, and that for $k \geqslant k_2$ the beam model can be used. In this paper it will be assumed that the solutions of the separate models are not interacting. In the next two sections the behavior of the string model and the beam model will be discussed, because these (partly) truncated models have not yet been treated in the literature.

4. The string model

Let us suppose $c_1^2 = 1$. In this section the equation

$$u_{\tau\tau} - u_{\xi\xi} = -\varepsilon u_\xi \cos\tau - 2\varepsilon(2+\sin\tau)u_{\xi\tau} - \varepsilon^2(2+\sin\tau)^2 u_{\xi\xi} \tag{13}$$

subject to the boundary conditions (6) will be studied.

The following ansatz will be used:

$$u(\xi,\tau) = \sum_{k=1}^{k_1} u_k(\tau)\sin(k\xi). \tag{14}$$

This leads to a system of coupled ODEs (see also Ref. [1] for the computations) for $k = 1, 2, \ldots, k_1$:

$$\frac{\mathrm{d}^2 u_k}{\mathrm{d}\tau^2} + k^2 u_k = \varepsilon {\sum_n}' \frac{2n}{2j+1}\left[u_n\cos\tau + 2(2+\sin\tau)\frac{\mathrm{d}u_n}{\mathrm{d}\tau}\right] + \varepsilon^2(2+\sin\tau)^2 k^2 u_k, \tag{15}$$

where ${\sum_n}' = \sum_{k=n-2j+1}\sum_{k=2j+1+n}\sum_{k=2j+1-n}\sum$. Without loss of generality it will be assumed that k_1 is even.

To analyze Eq. (15) a multiple time-scales perturbation method will be used. By introducing the slow time $\tau_1 = \varepsilon\tau$ it follows that

$$\frac{\mathrm{d}}{\mathrm{d}\tau} = \frac{\partial}{\partial\tau} + \varepsilon\frac{\partial}{\partial\tau_1}. \tag{16}$$

The function $u_k(t)$ may be sought in the following form:

$$u_k = u_k^{(0)} + \varepsilon u_k^{(1)} + \varepsilon^2 u_k^{(2)} + \ldots, \tag{17}$$

where $u_k^{(i)} = u_k^{(i)}(\tau, \tau_1)$.

Substituting Eqs. (16), (17) into Eq. (15) one obtains as $O(1)$ and as $O(\varepsilon)$-problem

$$\frac{\partial^2 u_k^{(0)}}{\partial\tau^2} + k^2 u_k^{(0)} = 0, \tag{18}$$

$$\frac{\partial^2 u_k^{(1)}}{\partial\tau^2} + k^2 u_k^{(1)} = -2\frac{\partial^2 u_k^{(0)}}{\partial\tau\partial\tau_1} + {\sum_n}' \frac{2n}{2j+1}\left[u_n^{(0)}\cos\tau + 2(1+\sin\tau)\frac{\partial u_n^{(0)}}{\partial\tau}\right], \tag{19}$$

respectively.

The solution of Eq. (18) is given by

$$u_k^{(0)} = A_k(\tau_1)\cos(k\tau) + B_k(\tau_1)\sin(k\tau). \tag{20}$$

The functions $A_k(\tau_1)$, $B_k(\tau_1)$ are defined such that no secular terms occur in Eq. (19). This leads to the following system of ODEs (see also Ref. [1]):

$$\frac{\mathrm{d}A_k}{\mathrm{d}\tau_1} = (k+1)B_{k+1} + (k-1)B_{k-1},$$

$$\frac{\mathrm{d}B_k}{\mathrm{d}\tau_1} = -(k+1)A_{k+1} - (k-1)A_{k-1}, \tag{21}$$

with $k = 1, 2, \ldots, k_1$, and $A_0 = B_0 \equiv 0$, $A_{k_1+1} = B_{k_1+1} \equiv 0$.

Assuming that the solutions of the system (21) can be written in the form

$$A_k(\tau_1) = C_k \mathrm{e}^{\lambda\tau_1}, \quad B_k(\tau_1) = D_k \mathrm{e}^{\lambda\tau_1}, \tag{22}$$

where λ, C_k, D_k are constants, it follows from Eqs. (21) and (22) that λ, C_k and D_k for $k = 1, 2, \ldots, k_1$ have to satisfy the following system of linear algebraic equations:

$$\lambda C_k = (k+1)D_{k+1} + (k-1)D_{k-1},$$

$$\lambda D_k = -(k+1)C_{k+1} - (k-1)C_{k-1}. \tag{23}$$

First the case $\lambda = 0$ is considered, and system (23) then implies

$$\begin{aligned} D_{2k} &= C_{2k} = 0, \\ D_{2k+1} &= (-1)^k \frac{D_1}{2k+1}, \\ C_{2k+1} &= (-1)^k \frac{C_1}{2k+1}, \end{aligned} \tag{24}$$

with $k = 1, 2, \ldots, k_1/2 - 1$.

For $\lambda \neq 0$ system (23) can be reduced to the following form:

$$\lambda D_k = -(k+1)C_{k+1} - (k-1)C_{k-1}, \tag{25}$$

$$-\lambda^2 C_k = (k-1)(k-2)C_{k-2} + 2k^2 C_k + (k+1)(k+2)C_{k+2}, \quad k = 1, 2, \ldots, k_1. \tag{26}$$

Eq. (26) can be studied separately for C_{2k} and C_{2k+1}. The case for odd indices will be studied. For even indices the results are similar.

Let us rewrite system (26) for odd indices as follows:

$$\frac{(k-1)(k-2)}{2k^2} C_{k-2} + \left(1 + \frac{\lambda^2}{2k^2}\right) C_k + \frac{(k+1)(k+2)}{2k^2} C_{k+2} = 0 \tag{27}$$

for $k = 1, 3, 5, \ldots, k_1 - 1$, and where C_{-1} and C_{k_1+1} are both zero.

The determinant of system (27) has the form

$$\det(\lambda^2) = \begin{vmatrix} \frac{1}{9} & 1 + \frac{\lambda^2}{2 \cdot 3^2} & 1\frac{1}{9} & 0 & 0 & \cdots \\ 0 & \frac{6}{25} & 1 + \frac{\lambda^2}{2 \cdot 5^2} & \frac{21}{25} & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \frac{(k_1-4)(k_1-5)}{2(k_1-3)^2} & 1 + \frac{\lambda^2}{2(k_1-3)^2} & \frac{(k_1-2)(k_1-1)}{2(k_1-3)^2} \\ \cdots & \cdots & \cdots & \cdots & \frac{(k_1-2)(k_1-3)}{2(k_1-1)^2} & 1 + \frac{\lambda^2}{2(k_1-1)^2} \end{vmatrix}. \tag{28}$$

The equation $\det(\lambda^2) = 0$ has as only real root $\lambda = 0$. All other roots λ are purely imaginary as has been shown in Ref. [1].

For the lower frequencies the string like approach is applicable, whereas for the higher frequencies we should look at a stretched beam approach and for a beam approach. So, for the lower frequencies this new approach indicates bounded solutions, whereas the old approach (see Ref. [1]) indicates unbounded solutions.

5. The beam model

Now the beam equation

$$u_{\tau\tau} + \varepsilon_2^2 u_{\xi\xi\xi\xi} = -\varepsilon u_\xi \cos(\tau) - 2\varepsilon(2 + \sin(\tau))u_{\xi\tau} - \varepsilon^2(2 + \sin(\tau))^2 u_{\xi\xi} \tag{29}$$

subject to the boundary conditions (6) and (7) will be studied.

Now the solution will be sought in the form

$$\sum_{k=k_2}^{\infty} u_k(\tau, \tau_1) \sin(k\xi)$$

and a multiple scale approach (16), (17) will be used. Then the following $O(1)$ and $O(\varepsilon)$ will be obtained:

$$\frac{\partial^2 u_k^{(0)}}{\partial \tau^2} + \varepsilon_2^2 k^4 u_k^{(0)} = 0, \tag{30}$$

$$\frac{\partial^2 u_k^{(1)}}{\partial \tau^2} + \varepsilon_2^2 k^4 u_k^{(1)} = -2\frac{\partial^2 u_k^{(0)}}{\partial \tau \partial \tau_1} + \sum_{n=k_2}^{\infty}{}'' \frac{nk}{n^2-k^2}\left[u_n^{(0)}\cos(\tau) + 8(2+\sin\tau)\frac{\partial u_n^{(0)}}{\partial \tau}\right], \tag{31}$$

respectively, with $k = k_2, k_2+1, k_2+3\ldots$. In $\sum_{n=k_2}''$ the summation is carried out only for $n \pm k$ odd.

Solutions of Eq. (30) have the form

$$u_k^{(0)} = A_k(\tau_1)\sin(\omega_k\tau) + B_k(\tau_1)\cos(\omega_k\tau),$$

where $\omega_k = \varepsilon_2 k^2$. Eq. (31) now becomes

$$\begin{aligned}\frac{\partial^2 u_k^{(1)}}{\partial \tau^2} + \omega_k^2 u_k^{(1)} = & -2\omega_k\left[\frac{dA_k}{d\tau_1}\cos(\omega_k\tau) - \frac{dB_k}{d\tau_1}\sin(\omega_k\tau)\right] \\ & + \sum_{n=k_2}{}'' \frac{nk}{n^2-k^2}\{4\cos(\tau)[A_n\sin(\omega_n\tau) + B_n\cos(\omega_n\tau)] \\ & + 8\omega_n(2+\sin(\tau))[A_n\cos(\omega_n\tau) - B_n\sin(\omega_n\tau)]\}.\end{aligned} \tag{32}$$

Internal resonances can take place in two cases (as has been shown in Ref. [2]):

$$\omega_k \pm \omega_p = 1, \quad k, p \geqslant k_2.$$

Let us suppose $\omega_{k_2+1} - \omega_{k_2} = 1$.

Then it follows from Ref. [2] that only the modes k_2+1 and k_2 are interacting, and that

$$\begin{aligned}A_{k_2}(\tau_1) &= -aB_{k_2+1}(0)\sin(\beta\tau_1) + A_{k_2}(0)\cos(\beta\tau_1),\\ B_{k_2}(\tau_1) &= aA_{k_2+1}(0)\sin(\beta\tau_1) + B_{k_2}(0)\cos(\beta\tau_1),\\ A_{k_2+1}(\tau_1) &= A_{k_2+1}(0)\cos(\beta\tau_1) - a^{-1}B_{k_2}(0)\sin(\beta\tau_1),\\ B_{k_2+1}(\tau_1) &= B_{k_2+1}(0)\cos(\beta\tau_1) + a^{-1}A_{k_2}(0)\sin(\beta\tau_1),\end{aligned} \tag{33}$$

where $a = \sqrt{(k_2+1)^2/k_2^2}$, $\beta = 2(2k_2^2 + 2k_2 + 1)/(3k_2(k_2+1))$. For $k \geqslant k_2+2$ it also follows from Ref. [2] that $A_k(\tau_1) = A_k(0)$ and $B_k(\tau_1) = B_k(0)$. For $\omega_{k_2+1} - \omega_{k_2} = 1$ it follows from Ref. [2] that the solutions are bounded (stable).

For the case $\omega_{k_2+1} + \omega_{k_2} = 1$ one can use the expression (33), where $\cos \to \cosh$, $\sin \to \sinh$ and $\beta = 2(2k_2+1)/(3k_2(k_2+1))$. In this case the solution of (29) can become unbounded (unstable).

For $k_1 + 1 \leqslant k \leqslant k_2 - 1$ one must use the stretched beam model—Eq. (5) with boundary conditions (6) and (7). This case has been completely analyzed in Ref. [2], that is why it is not repeated here.

6. Conclusion

In Refs. [1–3] it has been shown, that the infinite system of ODEs (21) (if one supposes $k_1 = \infty$) cannot be reduced. On the other hand, in engineering papers (see the references in Refs. [1–3]) only a few terms in the series (14) will usually be used. In what way can this contradiction be resolved?

It should be clear that the applicability of the different models is related to the frequencies (for the lower frequencies a string-like approach can be used, and for the higher frequencies stretched beam or beam-like approaches should be used). The different models still can (or should) be linked. The combination of models is new; and in the paper it is indicated how this method (of different models for different frequencies domains) can most likely be applied.

In mechanics usually a hierarchy of models can be used. In our case (depending on the frequency domain) there are string, stretched beam, beam, and 3D elasticity models. In turn, the applicability of a 3D model will be restricted by the atomic structure of the material. So, each model can only be used for a finite range of the parameters.

At the end of Section 3 it has explicitly been assumed that the string model for the modes $1 \leqslant k \leqslant k_1$, the stretched beam model for the modes $k_1+1 \leqslant k \leqslant k_2-1$, and the beam model for the modes $k \geqslant k_2$ are not interacting. However, by putting

$$u(\xi,\tau)=\sum_{n=1}^{\infty} u_n(\tau)\phi_n(\xi), \tag{34}$$

it will be clear that near the "boundaries" of validity of the different models the vibration modes certainly will interact. Observe that for the boundary value problems that are considered in this paper $\phi_n(\xi)$ is equal to $\sin(n\xi)$, and so $u_{\xi\xi} \sim n^2$ and $u_{\xi\xi\xi\xi} \sim n^4$. Based upon the aforementioned considerations the following approach will be proposed. For small values of n in Eq. (34) a string-like model should be used, that is, for $1 \leqslant n \leqslant n_1$ (and n_1 is of order 1)

$$u_{\tau\tau}-u_{\xi\xi}=\varepsilon f_1(\xi,\tau,u).$$

For $n \sim 1/\sqrt{\varepsilon}$, that is, for $n_1+1 \leqslant n \leqslant n_2$

$$u_{\tau\tau}-u_{\xi\xi}=\varepsilon f_2(\xi,\tau,u),$$

for $n \sim 1/\varepsilon$, that is, for $n_2+1 \leqslant n \leqslant n_3$ the stretched beam equation

$$u_{\tau\tau}-u_{\xi\xi}+\varepsilon^2 u_{\xi\xi\xi\xi}=\varepsilon f_3(\xi,\tau,u)$$

and so on, ending for instance with beam-like models such as

$$u_{\tau\tau}+\varepsilon^2 u_{\xi\xi\xi\xi}=f_4(\xi,\tau,u)$$

for $n \sim 1/\varepsilon\sqrt{\varepsilon}$ or larger.

One of the interesting difficulties for future research will be how to determine n_1, n_2, n_3, and so on. Moreover, it will be interesting to study the so-called system of (infinitely many) ODEs, and to answer the questions: Can the system be truncated? When the system can or cannot be truncated: what can be said about the stability of the solution of the system?

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